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UWAGA! Niektóre z powyższych wzorów zachodzą przy odpowiednich założeniach!

Podstawowe tożsamości trygonometryczne

- $\sin^2\alpha + \cos^2\alpha = 1$
- $\operatorname{tg}\alpha = \dfrac{\sin\alpha}{\cos\alpha}$
- $\operatorname{ctg}\alpha = \dfrac{\cos\alpha}{\sin\alpha}$
- $\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1$

Funkcje trygonometryczne podwojonego kąta

- $\sin 2\alpha = 2\sin\alpha\cos\alpha = \dfrac{2\operatorname{tg}\alpha}{1+\operatorname{tg}^2\alpha}$
- $\operatorname{tg} 2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha} = \dfrac{2}{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}$
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1$
- $\operatorname{ctg} 2\alpha = \dfrac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha} = \dfrac{\operatorname{ctg}\alpha - \operatorname{tg}\alpha}{2}$

Funkcje trygonometryczne sumy i różnicy kątów

- $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$
- $\operatorname{tg}(\alpha+\beta) = \dfrac{tg\,\alpha + tg\beta}{1 - tg\alpha tg\beta}$
- $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$
- $\operatorname{tg}(\alpha-\beta) = \dfrac{tg\,\alpha - tg\beta}{1 + tg\alpha tg\beta}$
- $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$
- $ctg(\alpha+\beta) = \dfrac{ctg\alpha ctg\beta - 1}{ctg\beta + ctg\alpha}$
- $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
- $ctg(\alpha-\beta) = \dfrac{ctg\alpha ctg\beta + 1}{ctg\beta - ctg\alpha}$

Sumy i różnice funkcji trygonometrycznych

- $\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
- $\operatorname{tg}\alpha + \operatorname{tg}\beta = \dfrac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$
- $\sin\alpha - \sin\beta = 2\cos\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
- $\operatorname{tg}\alpha - \operatorname{tg}\beta = \dfrac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$
- $\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$
- $\operatorname{ctg}\alpha + \operatorname{ctg}\beta = \dfrac{\sin(\beta+\alpha)}{\sin\alpha\sin\beta}$
- $\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$
- $\operatorname{ctg}\alpha - \operatorname{ctg}\beta = \dfrac{\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$

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Iloczyny funkcji trygonometrycznych

$\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$

$\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha-\beta) - \sin(\alpha+\beta)]$

$\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

Wybrane wzory redukcyjne funkcji trygonometrycznych

$\sin(90^\circ-\alpha) = \cos\alpha$

$\cos(90^\circ-\alpha) = \sin\alpha$

$\mathrm{tg}\,(90^\circ-\alpha) = \mathrm{ctg}\,\alpha$

$\mathrm{ctg}\,(90^\circ-\alpha) = \mathrm{tg}\,\alpha$

$\sin(90^\circ+\alpha) = \cos\alpha$

$\cos(90^\circ+\alpha) = -\sin\alpha$

$\mathrm{tg}\,(90^\circ+\alpha) = -\mathrm{ctg}\,\alpha$

$\mathrm{ctg}\,(90^\circ+\alpha) = -\mathrm{tg}\,\alpha$

$\sin(180^\circ-\alpha) = \sin\alpha$

$\cos(180^\circ-\alpha) = -\cos\alpha$

$\mathrm{tg}\,(180^\circ-\alpha) = -\mathrm{tg}\,\alpha$

$\mathrm{ctg}\,(180^\circ-\alpha) = -\mathrm{ctg}\,\alpha$

$\sin(180^\circ+\alpha) = -\sin\alpha$

$\cos(180^\circ+\alpha) = -\cos\alpha$

$\mathrm{tg}\,(180^\circ+\alpha) = \mathrm{tg}\,\alpha$

$\mathrm{ctg}\,(180^\circ+\alpha) = \mathrm{ctg}\,\alpha$

$\sin(270^\circ-\alpha) = -\cos\alpha$

$\cos(270^\circ-\alpha) = -\sin\alpha$

$\mathrm{tg}\,(270^\circ-\alpha) = \mathrm{ctg}\,\alpha$

$\mathrm{ctg}\,(270^\circ-\alpha) = \mathrm{tg}\,\alpha$

$\sin(270^\circ+\alpha) = -\cos\alpha$

$\cos(270^\circ+\alpha) = \sin\alpha$

$\mathrm{tg}\,(270^\circ+\alpha) = -\mathrm{ctg}\,\alpha$

$\mathrm{ctg}\,(270^\circ+\alpha) = -\mathrm{tg}\,\alpha$

$\sin(360^\circ-\alpha) = -\sin\alpha$

$\cos(360^\circ-\alpha) = \cos\alpha$

$\mathrm{tg}\,(360^\circ-\alpha) = -\mathrm{tg}\,\alpha$

$\mathrm{ctg}\,(360^\circ-\alpha) = -\mathrm{ctg}\,\alpha$

$\sin(360^\circ+\alpha) = \sin\alpha$

$\cos(360^\circ+\alpha) = \cos\alpha$

$\mathrm{tg}\,(360^\circ+\alpha) = \mathrm{tg}\,\alpha$

$\mathrm{ctg}\,(360^\circ+\alpha) = \mathrm{ctg}\,\alpha$

Tablica wartości funkcji trygonometrycznych dla wybranych kątów ostrych

α	$\sin\alpha$	$\cos\alpha$	$\mathrm{tg}\,\alpha$	$\mathrm{ctg}\,\alpha$
0°	0	1	0	nie istnieje
15°	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
75°	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$2+\sqrt{3}$	$2-\sqrt{3}$
90°	1	0	nie istnieje	0